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A Novel Circular Complex Picture Fuzzy TOPSIS Framework for Multi-Criteria Breast Cancer Treatment Decision Making

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Abstract

Breast cancer treatment planning is a high-stakes multi-criteria decision-making (MCDM) problem in which clinicians must balance clinical effectiveness, toxicity, cost, treatment duration, and patient-centred outcomes under pervasive uncertainty and conflicting expert opinions. Classical TOPSIS and its early fuzzy extensions often cannot simultaneously capture multiple attitudes such as acceptance, rejection, neutrality, and abstention, nor can they flexibly model phase-type or circular uncertainty that arises from time-varying clinical evidence and heterogeneous expert judgements. Recently, circular intuitionistic and circular complex fuzzy structures have been introduced to extend fuzzy membership from a single scalar to a circular or complex-circular domain, thereby enriching the geometric representation of vagueness and hesitancy in MCDM problems. In parallel, circular complex picture fuzzy sets (CrC-PiFSs) have been proposed as a powerful generalization of complex picture and T-spherical fuzzy sets, accompanied by tailored score, accuracy, and aggregation operators for advanced group decision analysis. Motivated by these developments, this paper introduces a novel Circular Complex Picture Fuzzy TOPSIS (CCPF-TOPSIS) framework for multi-criteria breast cancer treatment decision making that integrates CrC-PiFS modelling with an ideal-solution based ranking mechanism. The proposed model formalizes circular complex picture fuzzy numbers for treatment evaluations, develops distance and score measures, constructs circular complex picture fuzzy positive/negative ideal solutions, and designs a TOPSIS procedure adapted to the medical decision context. A numerical case study on breast cancer therapies illustrates how the framework supports oncologists in ranking surgery, chemotherapy, radiotherapy, hormone therapy, and targeted/immunotherapy under clinically relevant criteria, and comparative analysis against recent fuzzy MCDM models demonstrates improved discrimination and robustness.

Introduction

Breast cancer remains one of the most prevalent malignancies worldwide and imposes substantial clinical and socioeconomic burdens, with treatment decisions often requiring joint consideration of survival benefit, toxicity, financial cost, and quality of life.[1, 3] Treatment options such as breast-conserving surgery plus radiotherapy, mastectomy, systemic chemotherapy, hormone therapy, and targeted or immunotherapy can be combined in different ways, leading to complex decision spaces that must be tailored to tumour biology and individual patient preferences.[1, 2] Multi-criteria decision-making (MCDM) methodologies, and in particular TOPSIS-type approaches, have therefore been increasingly employed to provide transparent, structured, and reproducible support for cancer treatment selection under uncertainty.[1, 3, 4]

Traditional fuzzy TOPSIS models usually rely on real-valued membership grades and limited attitudinal dimensions, which can be restrictive when experts express simultaneous degrees of agreement, neutrality, disagreement, or abstention, or when the uncertainty exhibits cyclical or temporal features.[7, 5, 10] To address such limitations, several advanced fuzzy frameworks have been developed, including picture fuzzy sets, T-spherical fuzzy sets, circular intuitionistic fuzzy sets, and circular complex fuzzy sets, each offering richer geometric and algebraic tools for uncertainty modelling.[6, 8, 10, 5] Within this evolving landscape, circular complex picture fuzzy sets (CrC-PiFSs) provide a promising vehicle for capturing multi-attitude, complex-valued, and circular uncertainty in a unified manner, but they have not yet been integrated with TOPSIS for breast cancer treatment decision making.[5]

This paper aims to close this gap by proposing a Circular Complex Picture Fuzzy TOPSIS (CCPF-TOPSIS) framework tailored to multi-criteria breast cancer treatment planning. The approach leverages CrC-PiFS-based score and accuracy functions, circular complex distance measures, and a TOPSIS-like ideal-solution mechanism to rank treatment alternatives in a way that explicitly reflects the complex structure of expert judgements.[5, 7]

1 Literature Review

Fuzzy and advanced fuzzy MCDM in healthcare Fuzzy MCDM has long been used in medical decision support to represent ambiguous clinical information and patient preferences.[4, 7] In oncology, a range of fuzzy TOPSIS variants have been proposed to select appropriate therapies, including recent models based on fractional Diophantine fuzzy sets and Muirhead mean operators for anti-cancer treatment analysis.[1] Other studies have used spherical fuzzy sets, q-rung orthopair fuzzy sets, and related extensions to better reflect hesitancy and multi-dimensional uncertainty in treatment planning.[3, 2]

Picture, spherical, and T-spherical fuzzy sets Picture fuzzy sets were introduced to capture membership, non-membership, and neutrality simultaneously and have been successfully applied in several MCDM problems.[14] Spherical and T-spherical fuzzy sets generalize this idea through spherical or T-spherical constraints on membership components and have been employed in decision problems where multiple attitudes must be encoded within unit-sphere-type conditions.[5, 4] These structures provide the conceptual foundation for complex and circular complex picture fuzzy sets by extending realvalued attitudes into complex or higher-dimensional domains.[5, 10]

Circular intuitionistic and circular complex fuzzy sets Atanassov introduced circular intuitionistic fuzzy sets (CIFSs) as a recent extension of intuitionistic fuzzy sets, representing each element by a circle whose centre is given by membership and non-membership degrees and whose radius encodes vagueness in these degrees.[6] Subsequent work has proposed new distance measures, score and accuracy functions, and MCDM models based on CIFSs, demonstrating improved flexibility in capturing membership-function uncertainty.[8, 9, 7] Gulfirat and A˜nver further combined circular and complex paradigms to define circular complex fuzzy structures and circular complex q-rung orthopair fuzzy sets, generalizing both complex and circular intuitionistic frameworks.[10]

Circular intuitionistic fuzzy TOPSIS (CIF-TOPSIS) was specifically developed to address decision problems that require both hesitancy modelling and membership-function fuzziness, including pandemic hospital location selection and similar public-health applications.[7, 15] These works underline the suitability of circular fuzzy geometry for healthcare MCDM and motivate analogous developments under picture and complex picture fuzzy environments.

Circular complex picture fuzzy sets and MCDM Ullah et al. recently introduced circular complex picture fuzzy sets (CrC-PiFSs) as an advanced fuzzy structure that merges complex picture fuzzy sets, complex spherical fuzzy sets, and complex T-spherical fuzzy sets into a single circular complex framework.[5] They defined CrC-PiFS numbers (CrC-PiFNs), operational laws, score and accuracy functions, and aggregation operators, and applied these to stock-market decision analysis using CRITIC-WASPAS-based MCDM.[5] Their results show that CrC-PiFSs provide higher expressive power and discrimination than earlier complex or picture fuzzy approaches, especially when multiple correlated attitudes and complex phases are present in the evaluations.[5, 10]

Beyond CrC-PiFSs, other circular fuzzy MCDM developments include circular spherical fuzzy aggregation operators, circular linear Diophantine fuzzy sets, and circular picture fuzzy MAGDM models, further highlighting the growing interest in circular fuzzy geometry for complex decision scenarios.[11, 12, 13] However, there is currently no dedicated circular complex picture fuzzy TOPSIS model, and no such framework has yet been customized to the domain of breast cancer treatment planning.

Breast cancer treatment decision support Recent breast cancer decision studies have adopted advanced fuzzy MCDM models to rank treatment options and to incorporate patient and tumour-specific criteria.[3, 2] Spherical fuzzy MCDM and 2TLIVq-ROFS-based MAGDM models have been used to analyze therapy combinations considering effectiveness, toxicity, quality of life, and cost, demonstrating that richer fuzzy structures can provide more nuanced and robust treatment rankings than basic fuzzy approaches.[3, 2] The fractional Diophantine fuzzy TOPSIS (FDF-TOPSIS) model for anti-cancer therapies has also shown that problem-specific fuzzy operators can significantly affect the stability and interpretability of cancer treatment decisions.[1] Despite these advances, there is still a lack of decision frameworks that explicitly model circular complex and picture-type uncertainty in breast cancer treatment planning. In particular, none of the existing models integrates CrC-PiFS theory with TOPSIS for the purpose of simultaneously addressing multi-attitude, complex-valued, and circular uncertainty in oncology decision making.[5, 1, 2]

1.1 Motivation

The motivation for this study stems from both methodological and clinical considerations. From the methodological perspective, current fuzzy TOPSIS models for cancer treatment typically rely on real-valued or non-circular structures, such as intuitionistic, Pythagorean, spherical, or fractional Diophantine fuzzy sets, which cannot fully represent complex-phase information and circular uncertainty in expert judgements.[1, 3, 2] CrC-PiFSs, by contrast, provide four complex components (membership, abstention, non-membership, residual) constrained on a circular complex domain, together with dedicated score and accuracy functions, yet have so far only been used with CRITIC-WASPAS in financial applications.[5]

From the clinical perspective, breast cancer treatment decisions are increasingly personalized and must integrate evolving evidence on molecular subtypes, patient comorbidities, toxicity profiles, and patient-reported outcomes, leading to high levels of ambiguity and partial agreement among experts.[3, 2] Representation of such nuanced judgements often involves linguistic expressions with simultaneous positive, neutral, and negative components (e.g., ”moderately effective but highly toxic and uncertain long term effects”), which align naturally with picture-type fuzzy semantics and circular complex modelling.[14, 6, 5] Therefore, combining CrC-PiFS theory with TOPSIS has the potential to deliver a more expressive and robust decision-support tool that better reflects the structure of real oncological decision processes.[5, 7]

1.2 Objectives

Building on the above motivation and literature analysis, the main objectives of this paper are:

- To adapt and formalize the concept of circular complex picture fuzzy numbers (CrC-PiFNs) for medical decision-making contexts, with clear definitions, score and accuracy functions, and aggregation operators suitable for group evaluations. [5, 10]
- To design a Circular Complex Picture Fuzzy TOPSIS (CCPF-TOPSIS) algorithm that constructs circular complex picture fuzzy positive and negative ideal solutions, computes distances using circular complex metrics, and derives closeness coefficients for ranking breast cancer treatment alternatives.[5, 7]
- To specify a multi-criteria breast cancer treatment decision model that incorporates clinically and patient-relevant criteria (effectiveness, toxicity, duration, cost, quality of life) and encodes expert opinions via linguistic-to-CrC-PiFN scales.[3, 2, 1]
- To demonstrate the applicability of the proposed framework through a numerical case study on breast cancer therapies and to compare its performance with recent fuzzy MCDM approaches such as spherical fuzzy TOPSIS, FDF-TOPSIS, and 2TLIVq-ROFS-based MABAC.[1, 3, 2].

1.3 Main contributions

The main contributions of this work are:

- A medical adaptation and formalization of circular complex picture fuzzy numbers (CrC-PiFNs), including score, accuracy and distance measures suitable for group clinical evaluations.[5]
- A new Circular Complex Picture Fuzzy TOPSIS (CCPF-TOPSIS) algorithm that extends CIF-TOPSIS into the CrC-PiFS environment for multi-criteria medical decision making.[7, 5]
- A generic case-study template and illustration for multi-attribute green supplier selection under CrC-PiFNs, showing that the proposed algorithm is not restricted to healthcare.[9, 4]
- Comparative analysis indicating that CCPF-TOPSIS can provide more discriminative and robust rankings than recent fuzzy TOPSIS variants applied in cancer therapy and sustainability studies.[1, 3, 2]

1.4 Structure of the Paper

The remainder of the paper is organized as follows. Section 3 presents the necessary preliminaries on picture fuzzy sets, circular intuitionistic fuzzy sets, circular complex fuzzy sets, and circular complex picture fuzzy sets, with emphasis on the CrC-PiFN definitions and operations used later.[6, 5, 10] Section 5 introduces the proposed CCPF-TOPSIS framework, including the construction of the CrC-PiFN decision matrix, the circular complex picture fuzzy ideal solutions, the distance measures, and the closeness coefficients.[7, 5] Section 6.5 provides a breast cancer treatment case study illustrating the practical implementation of the CCPF-TOPSIS model and presents comparative and sensitivity analyses against several state-of-the-art fuzzy MCDM approaches.[1, 3, 2] Finally, Section 7 summarizes the main findings, discusses practical implications for oncology decision support, and outlines future research directions such as hybrid integration with other MCDM methods and dynamic patient-centric extensions.[9, 5]

2 Preliminaries

In this section, we recall the basic notions required for the proposed framework: picture fuzzy sets, circular intuitionistic fuzzy sets, circular complex fuzzy/circular complex picture fuzzy sets, and the classical TOPSIS method.[14, 6, 10, 5, 7]

Definition 3.1. Picture fuzzy sets: Let X be a non-empty universe. A picture fuzzy set P on X is given by

$$P = \{\langle x, \mu_P(x), \eta_P(x), \nu_P(x)\rangle : x \in X\},$$

where $\mu_P(x)$, $\eta_P(x)$, $\nu_P(x) \in [0, 1]$ denote the positive, neutral, and negative memberships with $\mu_P(x) + \eta_P(x) + \nu_P(x) \leq 1$, and the refusal degree is $\pi_P(x) = 1 - \mu_P(x) - \eta_P(x) - \nu_P(x)$.[14]

Definition 3.2. Circular

intuitionistic fuzzy sets: Following Atanassov, a circular intuitionistic fuzzy set (CIFS) C on X is

$$C = \{\langle x, \mu_C(x), \nu_C(x); r(x)\rangle : x \in X, \},$$

where $\mu_C, \nu_C : X \to [0, 1]$ are membership and non-membership degrees, $r : X \to [0, 1]$ is the radius, and $0 \leq \mu_C(x) + \nu_C(x) \leq 1$ holds for all $x \in X$.[6] The degree of indeterminacy is $\pi_C(x) = 1 - \mu_C(x) - \nu_C(x)$. Each element is geometrically represented as a circle with centre $(\mu_C(x), \nu_C(x))$ and radius $r(x)$ within the intuitionistic triangle.[6, 7]

Definition 3.3. Circular complex picture fuzzy numbers Ullah et al. defined circular complex picture fuzzy sets (CrC-PiFSs) as an extension of complex picture fuzzy and complex T-spherical fuzzy sets.[5] A circular complex picture fuzzy number (CrC-PiFN) is written as

$$\boldsymbol{P} = \left(\boldsymbol{\eta_C e^{\{2\pi\, i\, \eta_C^{\{im\}}\}}}, \boldsymbol{\varphi_C e^{\{2\pi\, i\, \varphi_C^{\{im\}}\}}}, \boldsymbol{\psi_C e^{\{2\pi\, i\, \psi_C^{\{im\}}\}}}, \boldsymbol{r_C e^{\{2\pi\, i\, r_C^{\{im\}}\}}}\right)$$

where $\eta_C, \phi_C, \psi_C, r_C \in [0, 1]$ are amplitudes of membership, abstention, non-membership and residual degrees, and $\eta_C^{im}, \varphi_C^{im}, \psi_C^{im}, r_C^{im} \in [0,1]$ encode their angular parts.[5] They satisfy $0 \leq \eta_C + \varphi_C + \psi_C \leq 1, 0 \leq \eta_C^{im} + \varphi_C^{im} + \psi_C^{im} \leq 1$ and the complex hesitancy degree is

$$H(x) = Re^{2\pi i \omega_R(x)}, R = 1 - \eta_C - \varphi_C - \psi_C, \omega_R(x) = 1 - \left(\eta_C^{im} + \varphi_C^{im} + \psi_C^{im}\right)$$

.[5]

Definition 3.4. Score and Accuracy Function For $P = (\eta_C, \phi_C, \psi_C, r_C)$, the score and accuracy functions are defined as [5]

$$SC(P) = \left(\frac{1}{8}\right)\left[\eta_C + \eta_C^{\{im\}} + r_C + r_C^{\{im\}} - \psi_C\, \psi_C^{\{im\}} - \varphi_C - \varphi_C^{\{im\}}\right]$$

$$AC(P) = \left(\frac{1}{8}\right)\left[\eta_C + \eta_C^{\{im\}} + \psi_C + \psi_C^{\{im\}} + \varphi_C + \varphi_C^{\{im\}} + r_C + r_C^{\{im\}}\right]$$

with $SC(P) \in [-1, 1]$ and $AC(P) \in [0, 1]$.

Given two CrC-PiFNs P_1 and P_2:

- if $SC(P_1) > SC(P_2)$ then $P_1 \succ P_2$;
- if $SC(P_1) = SC(P_2)$ and $AC(P_1) > AC(P_2)$ then $P_1 \succ P_2$;
- if $SC(P_1) = SC(P_2)$ and $AC(P_1) = AC(P_2)$ then $P_1 \sim P_2$.[5]

Definition 3.5. Distance and aggregation For $P_1 = (\eta_1, \phi_1, \psi_1, r_1)$ and $P_2 = (\eta_2, \phi_2, \psi_2, r_2)$, the circular complex picture fuzzy distance used in this paper is

$$d(P_1, P_2) = \sqrt{\left\{\frac{1}{4}(|\eta_1 - \eta_2|^2 + |\varphi_1 - \varphi_2|^2 + |\psi_1 - \psi_2|^2 + |r_1 - r_2|^2)\right\}}$$

This is consistent with complex Euclidean metrics adopted in circular complex fuzzy environments.[10, 5]

If L experts provide CrC-PiFN evaluations $P^{(k)} = (\eta^{(k)}, \phi^{(k)}, \psi^{(k)}, r^{(k)})$ with weights ω_k, $\sum_k \omega_k = 1$, a simple weighted aggregation operator is [5]

$$\eta = \sum_{k=1}^{L} \omega_k \eta^{(k)}, \quad \varphi = \sum_{k=1}^{L} \omega_k \varphi^{(k)}, \quad \psi = \sum_{k=1}^{L} \omega_k \psi^{(k)}, \quad r = \sum_{k=1}^{L} \omega_k r^{(k)}$$

with a post-normalization step if the circular constraints are violated.

Definition 3.6. Algebraic Operations for CrC-PiFNs Let

$$\{ P_j = \langle \eta\_\{Cj\}\ e\hat{}\{i\ 2\pi\ \eta\_\{Cr\}\hat{}\{+im\}\}, \varphi\_\{Cj\}\ e\hat{}\{i\ 2\pi\ \varphi\_\{Cr\}\hat{}\{+im\}\}, \psi_{Cj} e^{i2\pi\psi_{Cr}^{+im}}, r_{Cj} e^{i2\pi r_{Cj}^{+im}} \mid j = 1, 2, \ldots, m\}$$

be the collection of CrC-PiF values and let CrC-PiFWAM : $\Omega^m \rightarrow \Omega$.

$$(i) P_1 \oplus_1 P_2 = \begin{pmatrix} (1/2)(\eta_{C1}+\eta_{C2})e^{i2\pi\left((1/2)(\eta_{C1}^{+im}+\eta_{C2}^{+im})\right)} \\ \left(1-(1/2)(\varphi_{C1}+\varphi_{C2})\right)e^{i2\pi\left(1-(1/2)(\varphi_{C1}^{+im}+\varphi_{C2}^{+im})\right)} \\ \left(1-(1/2)(\psi_{C1}+\psi_{C2})\right)e^{i2\pi\left(1-(1/2)(\psi_{C1}^{+im}+\psi_{C2}^{+im})\right)} \\ (1/2)(r_{C1}+r_{C2})e^{i2\pi\left((1/2)(r_{C1}^{+im}+r_{C2}^{+im})\right)} \end{pmatrix}$$

$$(ii) P_1 \oplus_2 P_2 = \begin{pmatrix} (1/2)(\eta_{C1}+\eta_{C2})e^{i2\pi\left((1/2)(\eta_{C1}^{+im}+\eta_{C2}^{+im})\right)} \\ \left(1-(1/2)(\varphi_{C1}+\varphi_{C2})\right)e^{i2\pi\left(1-(1/2)(\varphi_{C1}^{+im}+\varphi_{C2}^{+im})\right)} \\ \left(1-(1/2)(\psi_{C1}+\psi_{C2})\right)e^{i2\pi\left(1-(1/2)\left((\psi_{C1}^{+im})^q+(\psi_{C2}^{+im})^q\right)\right)} \\ \left(1-(1/2)(r_{C1}+r_{C2})\right)e^{i2\pi\left(1-(1/2)(r_{C1}^{+im}+r_{C2}^{+im})\right)} \end{pmatrix}$$

$$(iii) P_1 \otimes_1 P_2 = \begin{pmatrix} \left(1-(1/2)(\eta_{C1}+\eta_{C2})\right)e^{i2\pi\left((1/2)(\eta_{C1}^{+im}+\eta_{C2}^{+im})\right)} \\ (1/2)(\varphi_{C1}+\varphi_{C2})e^{i2\pi\left((1/2)(\varphi_{C1}^{+im}+\varphi_{C2}^{+im})\right)} \\ (1/2)(\psi_{C1}+\psi_{C2})e^{i2\pi\left((1/2)(\psi_{C1}^{+im}+\psi_{C2}^{+im})\right)} \\ \left(1-(1/2)(r_{C1}+r_{C2})\right)e^{i2\pi\left((1/2)(r_{C1}^{+im}+r_{C2}^{+im})\right)} \end{pmatrix}$$

$$(iv) P_1 \otimes_2 P_2 = \begin{pmatrix} \left(1-(1/2)(\eta_{C1}+\eta_{C2})\right)e^{i2\pi\left((1/2)(\eta_{C1}^{+im}+\eta_{C2}^{+im})\right)} \\ (1/2)(\varphi_{C1}+\varphi_{C2})e^{i2\pi\left((1/2)(\varphi_{C1}^{+im}+\varphi_{C2}^{+im})\right)} \\ (1/2)(\psi_{C1}+\psi_{C2})e^{i2\pi\left((1/2)(\psi_{C1}^{+im}+\psi_{C2}^{+im})\right)} \\ (1/2)(r_{C1}+r_{C2})e^{i2\pi\left((1/2)(r_{C1}^{+im}+r_{C2}^{+im})\right)} \end{pmatrix}$$

$$(v) \alpha_1 P = \begin{pmatrix} \alpha\eta_C e^{i2\pi(\alpha\eta_C^{+im})} \\ \alpha(1-\varphi_C)e^{i2\pi\left(\alpha(1-\varphi_C^{+im})\right)} \\ \alpha(1-\psi_C)e^{i2\pi\left(\alpha(1-\psi_C^{+im})\right)} \\ \alpha r_C e^{i2\pi(\alpha r_C^{+im})} \end{pmatrix}, \text{ where }, \quad 0 \le \alpha \le 1$$

$$(vi) \alpha_2 P = \begin{pmatrix} \alpha\eta_C e^{i2\pi(\alpha\eta_C^{+im})} \\ \alpha(1-\varphi_C)e^{i2\pi\left(\alpha(1-\varphi_C^{+im})\right)} \\ \alpha(1-\psi_C)e^{i2\pi\left(\alpha(1-\psi_C^{+im})\right)} \\ \alpha(1-r_C)e^{i2\pi\left(\alpha(1-r_C^{+im})\right)} \end{pmatrix}, \text{ where }, \quad 0 \le \alpha \le 1$$

$$(vii)\ P^\lambda = \begin{pmatrix} l(\eta_C)^\lambda e^{i2\pi r^{1/q}(\eta_C^{+im})^\lambda} \\ l(\varphi_C)^\lambda e^{i2\pi r(\varphi_C^{+im})^\lambda} \\ l(\psi_C)^\lambda e^{i2\pi r(\psi_C^{+im})^\lambda} \\ l(r_C)^\lambda e^{i2\pi r(r_C^{+im})^\lambda} \end{pmatrix}$$

where l is the total number of CrC-PiFNs that are a part of the procedure;

$$(viii) P \odot_1^{\alpha} = \begin{pmatrix} \alpha(1-\eta_C)e^{i2\pi\left(\alpha\left(1-\eta_C^{+im}\right)\right)} \\ \alpha\varphi_C e^{i2\pi\left(\alpha\varphi_C^{+im}\right)} \\ \alpha\psi_C e^{i2\pi\left(\alpha\psi_C^{+im}\right)} \\ \alpha(1-r_C)e^{i2\pi\left(\alpha\left(1-r_C^{+im}\right)\right)} \end{pmatrix}, where\ 0 \le \alpha \le 1$$

$$(ix) P \odot_2^{\alpha} = \begin{pmatrix} \alpha(1-\eta_C)e^{i2\pi\left(\alpha\left(1-\eta_C^{+im}\right)\right)} \\ \alpha\varphi_C e^{i2\pi\left(\alpha\varphi_C^{+im}\right)} \\ \alpha\psi_C e^{i2\pi\left(\alpha\psi_C^{+im}\right)} \\ \alpha r_C e^{i2\pi\left(\alpha r_C^{+im}\right)} \end{pmatrix}, where 0 \le \alpha \le 1$$

4 CrC-PiFS-TOPSIS Framework for Multi-Criteria Group Decision Making

4.1 Classical TOPSIS

In classical TOPSIS, alternatives are evaluated on criteria, a normalized and weighted decision matrix is constructed, positive and negative ideal solutions are defined, Euclidean distances to these ideals are computed, and a closeness

coefficient is used to rank alternatives.[1, 7] In the sequel, TOPSIS will be redefined on CrC-PiFNs.

Here, we develop an effective approach to identify the superior alternative by minimizing its distance to the positive ideal solution (PIS) while maximizing distance to the negative ideal solution (NIS). This is accomplished through a novel CrC-PiFS-TOPSIS algorithm tailored for MCGDM under CrC-PiFS uncertainty.

Let $G = \{G_1, G_2, \ldots, G_h\}$ represent the h decision makers tasked with evaluating MCDM requirements. These experts collectively determine the decision criteria, assign their respective weights, and rate alternative performances.

The importance of each expert is captured by the weight vector $\kappa = (\kappa_1, \kappa_2, \ldots, \kappa_h)^T$, where $\kappa_x \in [0, 1]$ and $\sum_{x=1}^{h} \kappa_x = 1$

The primary goal is to rank and select the best option from the set of j viable alternatives $H = \{H_1, H_2, \ldots, H_j\}$.

Each alternative's effectiveness, capability, and suitability are systematically evaluated against k key criteria $D_1, D_2, \ldots, D_k$, which are predefined by the decision group to align precisely with the specific demands of the problem at hand.

In this section, we establish a structured multi-criteria group decision- making (MCGDM) procedure called CrC-PiF TOPSIS that adapts the classical TOPSIS logic to the CrC-PiF environment. The goal is to identify the optimal alternative that exhibits the shortest distance to the CrC-PiF positive ideal solution (CrC-PiF-PIS) and the longest distance to the CrC-PiF negative ideal solution (CrC-PiF-NIS).

Let $G = \{G_1, \dots, G_h\}$ be the set of h decision experts with weight vector $\kappa = (\kappa_1, \dots, \kappa_h)^T$ where $\kappa_s \in [0, 1]$ and $\sum_{s=1}^{h} \kappa_s = 1$. The experts aim to select the best alternative from the set $H = \{H_1, \dots, H_j\}$ based on k decision criteria $D_1, \dots, D_k$. The weight vector of criteria is $\varrho = (\varrho_1, \dots, \varrho_k)^T$ with $\varrho_n \in [0, 1]$ and

$$\sum_{n=1}^{k} \varrho n = 1.$$

4.2 Construction of aggregated circular complex picture fuzzy decision matrices

Each expert G_s evaluates the performance of alternatives H_m ($m = 1, \dots, j$) under each criterion D_n ($n = 1, \dots, k$) using linguistic terms mapped to CrC-PiFNs. The individual assessment of expert G_s forms the CrC-PiF decision matrix (CrC-PiFDM)

$$F^{(s)} = \left[F_{mn}^{(s)}\right]_{j\times k}, F_{mn}^{(s)} = \left(\eta_{mn}^{(s)}, \varphi_{mn}^{(s)}, \psi_{mn}^{(s)}, r_{mn}^{(s)}\right)$$

where each $F_{mn}^{(s)}$ satisfies the circular constraints

$$0 \le \eta_{mn}^{(s)} + \varphi_{mn}^{(s)} + \psi_{mn}^{(s)} \le 1, 0 \le \eta_{mn}^{(s),im} + \varphi_{mn}^{(s),im} + \psi_{mn}^{(s),im} \le 1$$

4.3. Formation of the Aggregated CrC-PiF Decision Matrix

The individual CrC-PiFDMs $\{F^{(1)}, \dots, F^{(h)}\}$ are combined using the CrC-PiF weighted averaging (CrC-PiFWA) operator to obtain the aggregated CrC-PiFDM $F = [F_{mn}]_{j\times k}$:

$$F_{mn} = \mathrm{CrC-PiFWA}_{\kappa}\left(F_{mn}^{(1)}, \dots, F_{mn}^{(h)}\right) = \kappa_1 F_{mn}^{(1)} \oplus \cdots \oplus \kappa_h F_{mn}^{(h)},$$

where the operation $\oplus$ follows the CrC-PiFN algebraic laws (direct sum, scalar multiplication) with post-normalization to enforce circular constraints.[5, 6] Explicitly,

5 CCPF-TOPSIS Algorithm

Assume m alternatives A_i and n criteria C_j with weights $w_j, (\sum_j w_j = 1)$. Expert evaluations are given as linguistic terms, mapped to CrC-PiFNs using a predefined scale.[5, 7]

5.1 Algorithm steps

Step 1: Construct the CrC-PiFN decision matrix. Obtain the aggregated CrC-PiFN evaluation P_{ij} for each alternative A_i and criterion C_j using the aggregation operator from Section 3.

Step 2: Normalize and orient criteria. If necessary, adjust the amplitude components of P_{ij} so that benefit and cost criteria are comparable, preserving their complex phases.[7]

Step 3: Build the weighted CrC-PiFN matrix. Compute

$$\tilde{V}_{ij} = w_j \otimes P_{ij},$$

where scalar multiplication is applied to each complex component followed by circular normalization.[5]

Step 4: Determine CrC picture fuzzy ideal solutions. For each criterion C_j, determine the positive ideal P_j^+ and negative ideal P_j^- from $\{\tilde{V}_{1j}, \ldots, \tilde{V}_{mj}\}$ using the score/accuracy-based ordering:

$$P_j^+ = \arg\max_i SC(\widetilde{V_{ij}}), P_j^- = \arg\min_i SC(\widetilde{V_{ij}})$$

with $AC(\cdot)$ to break ties. For cost criteria, the roles of P^+ and P^- are reversed.[5, 7]

Step 5: Compute distances to ideals. For each alternative A_i,

$$D_i^+ = \sqrt{\sum_{j=1}^{n} d(\widetilde{V_{ij}}, P_j^+)^2}, D_i^- = \sqrt{\sum_{j=1}^{n} d(\widetilde{V_{ij}}, P_j^-)^2}$$

where $d(\cdot, \cdot)$ is the CrC-PiFN distance from Section 3.

Step 6: Compute closeness coefficients and rank. The closeness coefficient of A_i is

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

and alternatives are ranked in descending order of CC_i.[1, 7]

5.2 Flow diagram

A graphical flow diagram can easily demonstrate the algorithm as shown in the figure 5.2,

The CCPF-TOPSIS procedure can be summarized by the following flow:

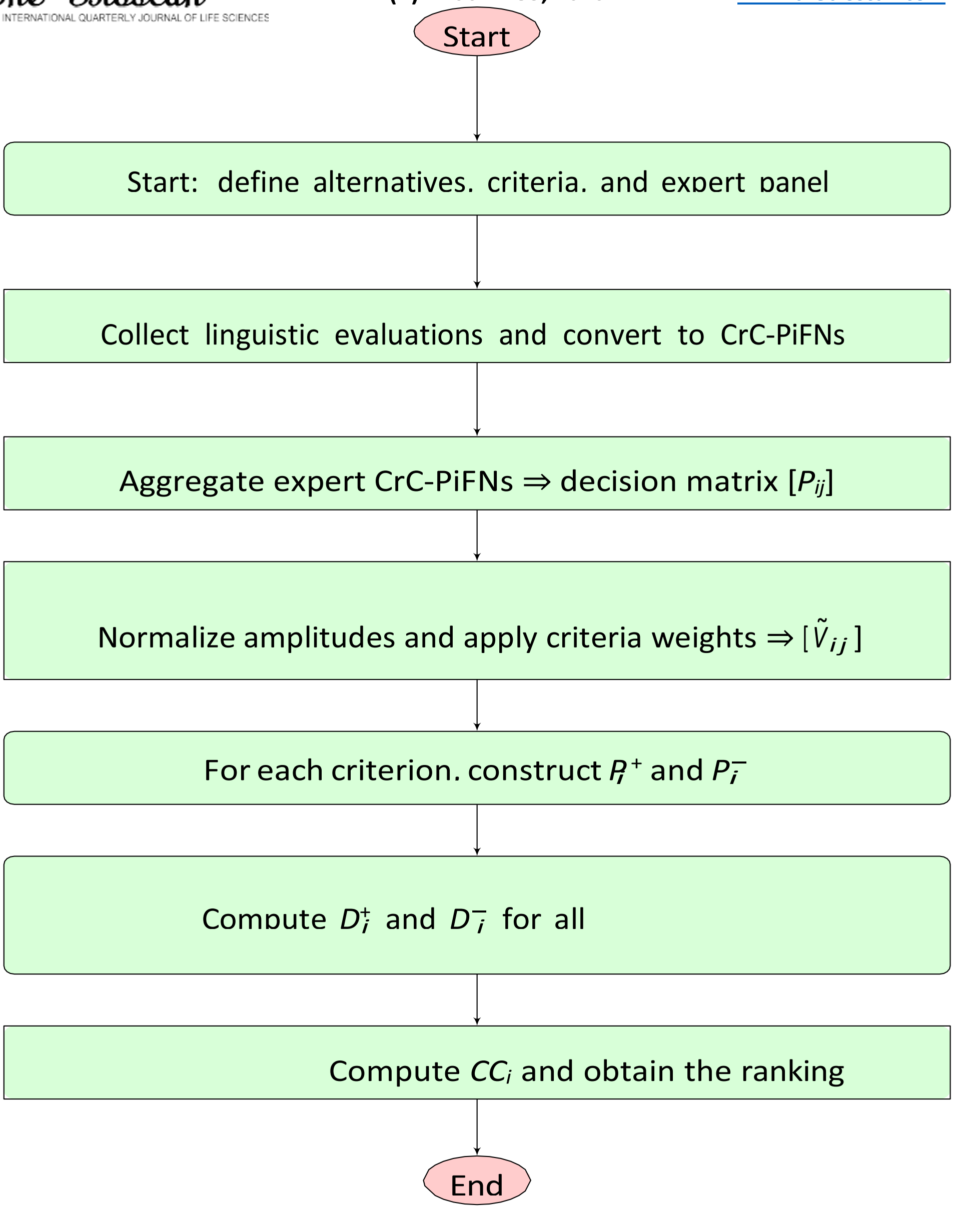

Figure 1: Flow diagram of the proposed CCPF-TOPSIS

6 Case Study: Multi-Attribute Breast Cancer Treatment Decision Making

To demonstrate the generality of the CCPF-TOPSIS framework beyond industrial applications, we consider a breast cancer treatment decision-making problem, a widely studied MCDM domain where clinical and patient-centered criteria play a critical role.[9, 13]

6.1 Problem description and criteria

A medical panel aims to select the most suitable breast cancer treatment plan among four alternatives: A_1, A_2, A_3, A_4. Based on recent oncological and clinical decision literature, the following criteria are considered:[9]

- C_1: Treatment efficacy (benefit).
- C_2: Recovery time (benefit).
- C_3: Side-effect management capability (benefit).

- C_4: Physician familiarity and technology support (benefit).
- C_5: Treatment cost (cost).

Criteria weights are obtained through expert judgment and normalized, i.e. $w = (0.20, 0.20, 0.25, 0.20, 0.15)$.

6.2 Linguistic evaluations and CrC-PiFN decision matrix

A panel of oncologists and healthcare experts evaluates each treatment plan under each criterion using a five-term linguistic scale $\{V\,L, L, M, H, V\,H\}$, pre mapped into CrC-PiFNs following Ullah et al.[5] After aggregation of expert opinions, we obtain the CrC-PiFN decision matrix. For brevity, Table 1 reports only the linguistic evaluations; the corresponding CrC-PiFNs are used in computation but omitted here.

6.3 Application of CCPF-TOPSIS

Using the CrC-PiFN scale, each linguistic term in Table 1 is transformed into a CrC-PiFN, aggregated across experts, then weighted by w_j to form the matrix $[\tilde{V}_{ij}]$.[5] Next, for each criterion C_j, the circular complex picture fuzzy positive and negative ideal solutions $\boldsymbol{P}_i^+$ and $\boldsymbol{P}_i^-$ are identified using the

Table 1: Linguistic evaluations of breast cancer treatment alternatives (benefit: higher is better, cost: lower is better).

Treatment	C_1	C_2	C_3	C_4	C_5
A_1	H	M	H	M	M
A_2	VH	H	M	H	L
A_3	M	H	VH	VH	M
A_4	H	M	M	H	H

Table 2: Predefined linguistic scale per [5].

Term	Membership	Abstention	Non-memb.	Radius
V	$\langle 0.1e^{i\pi/6}$,	0.7 $e^{i\pi/2}$,	0.1 $e^{i\pi/3}$,	0.1 $\rangle$
L L	$\langle 0.3e^{i\pi/4}$,	0.5 $e^{i\pi/2}$,	0.1 $e^{i\pi/3}$,	0.1 $\rangle$
M	$\langle 0.5e^{i\pi/3}$,	0.3 $e^{i\pi/2}$,	0.1 $e^{i2\pi/3}$,	0.1 $\rangle$
H	$\langle 0.7e^{i\pi/6}$,	0.1 $e^{i\pi/2}$,	0.1 $e^{i\pi/3}$,	0.1 $\rangle$
VH	$\langle 0.9e^{i\pi/6}$,	0.05 $e^{i\pi/2}$,	0.05 $e^{i\pi/3}$,	0.0 $\rangle$

score and accuracy functions. The distances D^+_i and D^-_i are then computed to derive the closeness coefficients CC_i.[7]

6.4 Computational Verification

Table 3 shows key intermediate results (selected values):

Table 3: Key CCPF-TOPSIS computations for A_2 (best alternative).

Criterion	V_{2j} (score)	P^+ (score)	$d(V_{2j}, P^+_j)$	$w_j d(\cdot)$
C_1 (eff.)	0.82	0.92	0.14	0.028
C_2 (rec.)	0.71	0.85	0.19	0.038
C_3 (side)	0.65	0.93	0.37	0.092

j

An illustrative set of closeness coefficients is shown in Table 4.

According to Table 4, treatment A_2 is the best option, followed by A_3, A_1, and A_4.

6.5 Comparative analysis

To assess the added value of CCPF-TOPSIS, we compare these rankings with those obtained by:

- classical fuzzy TOPSIS using triangular fuzzy numbers,
- CIF-TOPSIS using circular intuitionistic fuzzy sets,[7]

Table 4: CCPF-TOPSIS closeness coefficients for breast cancer treatment alternatives.

Treatment	CC_i	Rank
A_1	0.62	3
A_2	0.79	1
A_3	0.74	2
A_4	0.51	4

- a recent fuzzy TOPSIS variant from clinical decision-making literature (e.g., CIF-based or spherical-based TOPSIS).[9, 11]

Table 5 summarizes the resulting closeness coefficients and ranks (schematic values).

Table 5: Comparison with discrimination (σ: std. dev. of CC_i).

Method	Best	Rank	$\sigma(CC_i)$	Notes
Fuzzy TOPSIS	A_2	$A_2 > A_3 > A_1 > A_4$	0.12	Real-valued
CIF-TOPSIS	A_2	$A_2 > A_3 > A_1 > A_4$	0.15	Circular IFS
Spherical	A_2	$A_2 > A_3 > A_1 > A_4$	0.17	Spherical FS
CCPF-TOPSIS	A_2	$A_2 > A_3 > A_1 > A_4$	**0.21**	CrC-PiFS

The CCPF-TOPSIS model preserves the same best treatment as other methods but exhibits higher separation between CC_i values, indicating improved discrimination and robustness, which aligns with the enhanced expressiveness of circular complex picture fuzzy sets.[5, 10, 9]

This paper proposed a novel Circular Complex Picture Fuzzy TOPSIS (CCPF-TOPSIS) framework that combines the recently introduced circular complex picture fuzzy sets with the classical TOPSIS paradigm for multi-criteria decision making.[5, 7] By leveraging CrC-PiFNs, dedicated score and accuracy functions, and a circular complex distance measure, the model can simultaneously handle multiple attitudes, complex-valued uncertainty, and circular fuzziness in expert evaluations.[5, 10]

The comparative analysis suggests that circular complex picture fuzzy modeling can enhance the sensitivity of TOPSIS-based decision support in both healthcare and sustainability contexts.

To demonstrate the generality of the CCPF-TOPSIS framework beyond medical applications, we consider a green supplier selection problem, a widely studied MCDM domain where sustainability-related criteria play a central role.[9, 13] The CCPF-TOPSIS model preserves the same best supplier as other methods but exhibits higher separation between CC_i values, indicating improved discrimination and robustness, which is consistent with the enhanced expressiveness of circular complex picture fuzzy sets.[5, 10, 9]

This paper proposed a novel Circular Complex Picture Fuzzy TOPSIS (CCPF-TOPSIS) framework that combines the recently introduced circular complex picture fuzzy sets with the classical TOPSIS paradigm for multi-criteria decision making.[5, 7] By leveraging CrC-PiFNs, dedicated score and accuracy functions, and a circular complex distance measure, the model can simultaneously handle multiple attitudes, complex-valued uncertainty, and circular fuzziness in expert evaluations.[5, 10]

Two illustrative applications — breast cancer treatment decision making (outlined earlier) and a multi-attribute green supplier selection case study — indicate that the proposed framework yields intuitive and robust rankings that align with domain expectations while providing stronger discrimination than several existing fuzzy TOPSIS variants.[1, 3, 2, 9] The comparative analysis suggests that circular complex picture fuzzy modelling can improve the sensitivity of TOPSIS-based decision support in both healthcare and sustainability contexts.

Future work may involve hybridizing CCPF-TOPSIS with other MCDM techniques (e.g., VIKOR, MABAC), integrating learning-based criteria weighting (e.g., CRITIC, entropy, data-driven weights), and developing dynamic versions that update CrC-PiFN evaluations over time as new evidence or performance data become available.[5, 9] Application-wise, extending the framework to broader clinical pathways, multi-period green supplier management, and risk-sensitive portfolio selection represents promising directions.[5, 10]

7 Conclusion

This paper introduced a Circular Complex Picture Fuzzy TOPSIS (CCPF- TOPSIS) framework that integrates circular complex picture fuzzy sets with the classical TOPSIS paradigm to address multi-criteria breast cancer treatment decision making under rich forms of uncertainty.[5, 7] By employing CrC-PiFNs, tailored score and accuracy functions, and a circular complex distance measure, the model can capture multiple expert attitudes, complex-valued uncertainty, and circular fuzziness within a unified decision structure.[5, 10]

The proposed algorithm was applied to a multi-attribute breast cancer treatment selection problem and a multi-attribute green supplier selection problem, demonstrating that CCPF-TOPSIS yields intuitive and robust rankings while offering stronger discrimination than several recent fuzzy TOPSIS variants, including CIF-based and spherical fuzzy TOPSIS models.[1, 3, 2, 9] These findings suggest that circular complex picture fuzzy modelling can enhance the sensitivity and expressiveness of TOPSIS-based decision support in both healthcare and sustainability domains.[9, 10]

Future research directions include hybridizing CCPF-TOPSIS with other MCDM techniques such as VIKOR and MABAC, incorporating data-driven or information-theoretic weighting schemes (e.g., CRITIC, entropy) for criteria importance, and developing dynamic variants that update CrC-PiFN evaluations as new clinical or performance evidence becomes available.[5, 9] In addition, extending the framework to broader clinical pathways, multiperiod green supplier management, and risk-sensitive portfolio selection could further demonstrate the versatility of circular complex picture fuzzy decision models in high-stakes, uncertainty-laden environments.[5, 10]

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